Improving regularity of Delaunay Triangulation of Khuzestan Bidbland Dome via Circle Packing Algorithm and Genetic Optimization

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ABSTRACT

Professor Nooshin et al. defined structural regularity indicators quantitatively three decades ago and used various methods for improving regularity of some structures, such as stepping projection, sphere packing concept, recursive techniques, and etc, to create a special category of space structures (domes and flats) and grid shells that were designed in a multifaceted manner, and depending on the type of project, they used one or more of these indicators to analyze and compare the regularity of the structures The regularity studied in this research is related to increase of equal lengths or almost equal lengths, the reduction the count of elements with different lengths, the reduction of the number of length intervals and different shapes of faces while maintaining the geometric form. The aim of this research is to study the previous regularity indicators, investigate and analyze the improving regularity of Delaunay triangulation obtained from the points of the single layer dome of Bidboland Khuzestan with the proposed algorithm: using the circle packing algorithm and genetic optimization (using the minimization of different regularity indicators) to reach a suitable pattern. In order to analysis the improving regularity. we have presented algorithms for calculating the regularity indicators, we have calculated the numerical results of these indices and we have suggested a general definition for regularity degree. we show that each of these indices alone are not criteria for measuring and comparing the regularity of two structures. The results of this research are effective in improving the regularity of structures, form finding and creating the optimal design of a structure.

KEYWORDS

Dome, Improving Regularity, Delanauy Triangulation, Genetic Optimization, Circle Packing Algorithm.

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1. Introduction

Referring to the report on "Current Status of Space Structures" published by the International Society of Shells and Space Structures (IASS) in 1984, A space structure can be considered as a structural system consisting of linear members and their arrangement is such that loads are transferred in three dimensions [1]. Space structures can be flat, barrel, cylinder, parabola, dome, freeform or a combination of these in terms of geometrical structure. The orientation of the members at the place of connection is very variable, which causes a great variety in the size of the elements and angles, therefore, it is always tried to design multiple elements in each structure in a few limited types, so that it can save time and money by accumulating parts [2-3]. Regularity and Improving Regularity of structures as a process of optimization has always been the focus of researchers for the purposes of optimal design, cost reduction, and aesthetics. There are different methods of improving the regularity such as stepping projection [4],traviation process[5], sphere packing concept [6], recursive techniques [7-9], and etc of a series of structures (flat and dome). To evaluate and compare the regularity of the structures, Proffesor Nooshin et al. the quantitatively defined the numerical indicators for measuring regularity. These indicators are known as regularity measuring meters or geometric parameters or degrees of regularity. These indicators along with source are introduced in Table 1. Another useful idea that is the concept of 'length profile chart' (LEP chart). A LEP chart gives a visual representation of the degree of scatter of the member lengths in a configuration [5]. Until now, a single definition for the geometric regularity of a structure and its regularity index has not been stated and they are different according to different goals. Although The consensus is that a high percentage of structural elements are the same or close to each other, and the ideal is that all elements have the same length [6,10].

Table 1-Regularity Geometric Indicators

Variance [5] $S^2 = \frac{\sum (Li - Lav)^2}{n}$	Shape Ratio [5] $Sr = \frac{\theta_{\max}}{\theta_{\min}}$	Length Ratio (r) [5] $r = \frac{L_{\max}}{L_{\min}}$
Length Deviation [4] $cv = \frac{S}{Lav} = \frac{\sqrt{\frac{\sum (Li - Lav)^2}{n}}}{Lav}$	Standard Deviation [10] $S = \sqrt{\frac{\sum (Li - Lav)^2}{n}}$	Variation Range [10] $D = L_{\max} - L_{\min}$

In our research for a case study we have chosen the single-layer dome of the Bidboland Persian Gulf gas refinery building, which is located 32km west of Shahid Behbahan in Khuzestan province in Iran. This structure was designed by the engineers of Ofogh Noor Space Structures Company in Rhino software (Figure 1). The diameter of this dome is about 8.5 meters and the height of the dome is about 1.3 meters and contains 57 points.

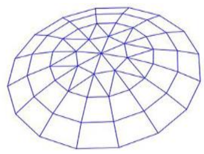


Figure 1. Khuzestan Bidboland Dome drawn in Rhino software, Source: Ofogh Noor Space Structure Company

The regularity investigated in the literature of this research refers to the increase of elements with the equal or almost the equal length, the reduction of the number of length intervals and the reduction of different shapes of the faces. In this research, for the first time, we analyze and investigate geometric regularity of the Delaunay triangulation using the minimization of measurement errors and the proposed algorithm: using the circle packing algorithm and genetic optimization with 57 input points while maintaining the geometry. Genetic optimization was performed using the galapagos plugin with default settings [11]. Also, we designed algorithms to calculate the regularity indicators and the percentage of different sides and angles of the structure based on the definitions of the regularity indices of previous researchers' studies, and we propose a general definition for the regularity indicator and we show that the regularity indicators alone are not a decision criterion for comparing and evaluating the regularity of two structures. In the case of indicators that take zero value in the state of complete regularity of the structure, the error value is equal to |regularity index| and if it is equal to 1 in the state of complete regularity of the structure, the amount of error is equal to |regularity index-1|. In this research, the length of the elements between 1000-1100 mm, 1100-1200 mm and etc, each shows a length type (length interval). Elements less than 1000 mm are considered as one length interval, and if the number of elements in one length interval is 2 or less, we don't consider it as a type, it is considered in the next type. In order to change the position of the points while maintaining the geometry of the dome, first the surface of the points is designed parametrically [12]. Then Delaunay triangulation has been done. The limitation of the problem is that the initial points of the structure are only allowed to move on the surface drawn through these points. We designed algorithms to calculate the indices of geometric regularity and the percentage of different sides and angles of the structure.

2. Methodology

The research process in this research is algorithmic, applied, computational, laboratory and analytical, and the stages of the research are:

1- Providing algorithms for calculating regularity indices and calculating the percentage of different sides and angles of the structure (Figure 2)
2- Designing the surface from the points of the structure parametrically (Figure 3)
3- Creating Delany triangulation (Figure 3)
4- Create circle packing algorithm (Figure 4)
5- Using genetic optimization (Galapagos plugin) with the aim of minimum error measurements
6- Calculation regularity indicators of the structures resulting from the implementation of step 5
7- Propose the general definition for reguarity index
8- Analysis of improving the regularity of dome

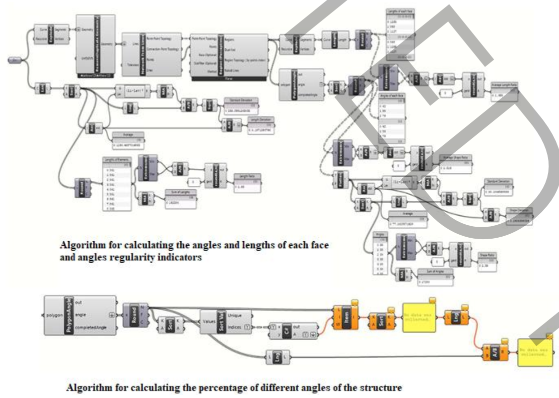


Figure 2. Step 1 algorithm, source :Authors

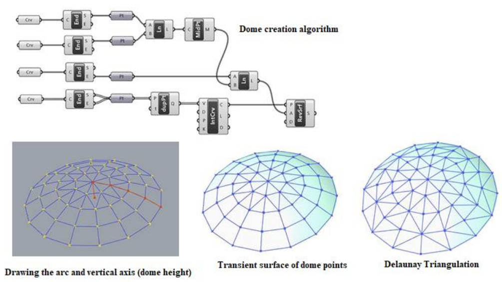


Figure 3. Algorithm steps 2 and 3, source: Authors

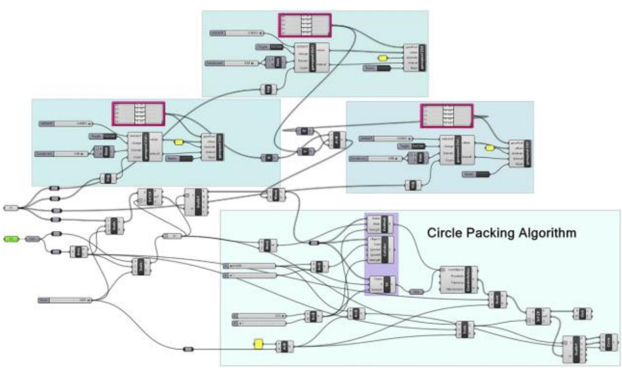
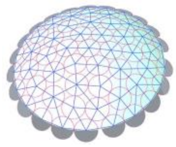
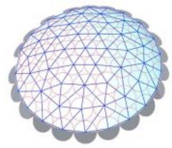
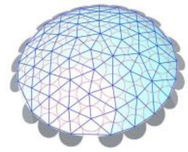
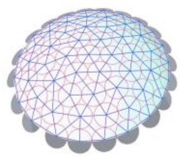
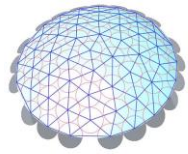
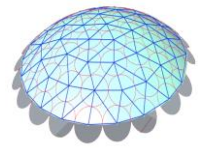
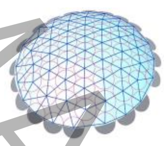
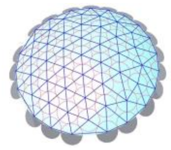


Figure 4. Circle Packing Algorithm, Source: Authors

3.Results and Discussion

The results of triangulation after genetic optimization are presented in figures 12-20.

Table 2- Results and Discussion

 Figure 5. Goal Function Min |1-r| Length types= 6	 Figure 6. Goal Function Min $C.V_{Li}$ Length types=6	 Figure 7. Goal Function Min |1-Area Ratio| Length types=7
 Figure 8. Goal Function Min |1-Sr| Length types=6	 Figure 9. Goal Function Min |1-AverageSr| Length types=8	 Figure 10. Goal Function Min $C.V_{areas}$ Length types=7
 Figure 11. Goal Function Min $S_{\theta i}$ Length types=7	 Figure 12. Goal Function Min $S^2_{\theta i}$ Length types=6	 Figure 13. Goal Function Min $C.V_{\theta i}$ Length types=6

We calculated all the length, angle and area indices of regularity for the above structures that be shown in Persian paper.

2.1 Proposed definition of Regularity index

According to researchers, the definition of regularity index is like the definition of beauty in architecture, which is different depending on different goals [4,6]. A suitable index for regularity is an index that does not change with the change of the symmetric scale. In this research, these indicators are: Length ratio, shape ratio, area ratio, average shape ratio, coefficient of variation of sides, variance of angles, standard deviation of angles, coefficient of variation of angles and coefficient of variation of area. Also, in addition to the above definitions, other regularity indices can also be defined, such as the multiplication of the length ratio by the shape ratio or their sum, or the multiplication of two regularity indices, etc. For the first time, we propose a general definition of the regularity indicator based on the relation (1) in which each one r_i **'s** represents one of the above-mentioned regularity indicators and each one can be a number between zero and one and their sum is one, or be determined according to the user's taste.

$$\Pr = a_1 r_1 + a_2 r_2 + \ldots + a_n r_n \quad (1)$$

4.Conclusions

In all triangulations, relation (2) is valid.

1- The number of points + the number of sides = the number of sides (2)

1- By Considering the area ratio as an regularity indicator, structure Figure 8, which is obtained with the aim of minimization|1- Sr|, it has a smaller value than other structures, so it has more regularity than other structures.

2-By Considering r and Sr as an regularity indicator, structure Figure 7, which is obtained with the aim of minimization|1- Area Ratio|, it has a smaller value than other structures, so it has more regularity than other structures.

3- By Considering each of the Sr, $C.V_{L_i}$, $C.V_{\theta i}$ and $S_{\theta i}$ as an regularity indicator, structure Figure 11, which is obtained with the aim of minimization $S_{\theta i}$, it has a smaller value than other structures, so it has more regularity than other structures.

4- By Considering the $C.V_{areas}$ as an regularity indicator, structure Figure 10, which is obtained with the aim of minimization $C.V_{areas}$ it has a smaller value than other structures, so it has more regularity than other structures.

5- The number of length types of structures Bidboland dome and dalanauy Bhdboland dome is the least, which simplifies assembly, assembly is a part of the work of engineers, and it can’t be said definitively whether these two structures have less weight than other structures. From the viewer's point of view, it is expected that these two structures have more regularity than other structures, while it is not consistent with the numerical results of the regularity indices.

The proposed definition shows that with different values of a_i 's, one of the structures case study and No. 5 to 12 will have more regularity, which do not look regular from a visual point of view, and a unique answer is not obtained. Therefore, it is necessary to improve the definition of regularity index in order to definitively decide which structure has more geometric regularity. This matter requires a lot of research and experiments, and a research team of researchers, structural engineers and mathematicians is needed to be able to provide a single definition for the regularity index.

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